**1.sol** Another example of a result that can neither be proved nor disproved by ZFC Axioms

**Hilbert’s tenth problem**

In Number Theory Diophantine Equations are those polynomials with integer coefficients

with finite number of unknowns

Hilbert’s tenth problem is “Finding an Algorithm (A standard procedure) to determine

whether the given Diophantine Equation has integral roots or not”

For example P(x , y) = 3x2 + 2y2  for his Diophantine Equation we need to find

P(k1 , k2) = 3(k1)2 + 2(k2)2 = 0 such that k1 , k2 ∈ Z

We know that no solution exist for P(x , y) = 0 as always x2 ≥ 0

But for problem like P(x , y , z) = 3x2 − 2xy – y2z – 7 = 0 to find solution we have to verify

many cases to find a perfect solution like x =1, y = 2, z = −2 and conclude equation has a

solution. We cannot prove whether this problem has integral solutions or not using ZFC’S

and Axiom of choices. We cannot make any algorithm which verifies the Diophantine

equation for all integers and finds integral solution exists or not. It would be like a forever

loop and it cannot verify for all values. Therefore, we cannot say whether Hilbert’s tenth

problem can be solved or no using ZFC’s.

Hence, Hilbert’s tenth problem in independent of ZFC’S i.e., ZFC’S won’t help in

solving the problem

Later using some other new axioms from Axiomatic set theory **Yuri Matiyasevich** has

concluded Hilbert’s tenth problem cannot be solved i.e., No algorithm can be made which

efficient solves and decides the possibility of solution for a Diophantine Equation.